The Search Costs of Inflation

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PRELIMINARY

Abstract

What are the costs of inflation in the labor market? When wages are set nominally, inflation leads to reduced purchasing power, which prompts workers to search for other jobs in order to adjust their real wages; this search is costly. We quantify these costs in a model of on-the-job search, where search effort responds endogenously to unexpected inflation. Unexpected inflation erodes the value of a match to a worker through real wage losses and larger search costs. The real wage loss is absorbed as a benefit to firms, whereas the cost of search is a net aggregate cost of inflation.

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1 Introduction

What is the cost of inflation? In the New Keynesian model, inflation creates dispersion in the price of identical goods distorting relative price signals and leading to inefficient allocations, but the empirical evidence for these costs is limited (Nakamura et al. 2018). Workers' dislike of inflation due to a perceived loss of purchasing power is another potential cost proposed by Shiller 1997 and considered again more recently (Stancheva 2024, Pilossoph and Ryngaert 2023, Hajdini et al. 2022, Jain, Kostyshyna, and Zhang 2022). In Pilossoph and Ryngaert 2023, we argued that workers respond to such losses with on-the-job search. This paper shows that search costs constitute the primary aggregate cost of inflation in the labor market.

In order to consider the welfare consequences of unexpected, repeated inflationary shocks, we extend the model of Postel-Vinay and Robin 2002 to include endogenous search effort and a price level with deterministic trend inflation. In our environment, wage contracts adjust with trend inflation. Yet, unexpected inflationary shocks have redistributive consequences: due to a two-sided lack of commitment, the nominal wages of incumbent workers only adjust through credible outside offers. Inflation therefor reduces the real wages of incumbent workers - transfering this surplus to employers as profits. As workers are able to select optimal search effort to control the likelihood of obtaining an outside offer, this reduction in real wages induces them to increase their search effort at a cost. This has two key implications. First, the redistributive role of unexpected inflation has allocative consequences, as its influence on search effort choices moves workers up the job ladder. Second, as the search costs incurred by workers are a form of deadweight loss, the redistributive role of inflation incurs aggregate welfare losses. Together, these mechanisms determine the aggregate welfare effects of unexpected inflationary shocks, which may have a net positive or net negative effect that depends on the structure of the labor market.

These mechanisms follow from the structure of wage determination in our model. The foundation is a two-sided lack of commitment, in which the nominal wage is fixed for the duration of the match unless either party has a credible threat which induces the nominal wage to change. This implies that, conditional on the nominal wage being fixed, real wages move in response to movements in the price level. Wages are negotiated in a new contract between an employer and employee when (i) the worker gets an outside offer which allows them to bargain an increased wage at their current firm - an outcome that depends on the worker's real wage, (ii) the worker receives an outside offer from a more productive company, resulting in a jobto-job transition - an outcome which is independent of the worker's real wage, (iii) inflation induces a large enough reduction in the worker's purchasing power that the net value to the worker is negative. The key implication is that the nominal rigidity present in this system of wage determination results in workers facing real wage cuts absent an outside offer, which then prompts them to increase their search effort. This increases the likelihood of job-to-job transitions, which may reallocate workers up the job ladder, but it also induces deadweight loss by increasing search costs.

We use our model to provide a quantitative estimate of the welfare consequences of a large unexpected inflationary shock that lasts for three years and that is similar in magnitude to the shock experienced in the early 1970s. To do so, we calibrate our model to match key features of the pre-COVID US labor market. We feed an identical, repeated shock into the model for a three year peroid. We simulate the flow value of output of each worker, netting out search costs, in each period after the shock commences, computing the average present discounted value of flow output in the first period of the shock, providing an aggregate welfare measure that we compare against a baseline economy without the shock. The lifetime welfare losses experienced by agents in the shocked economy amount to 0.14 percent of the PDV of agents in the baseline economy. We find that the reallocative channel of unexpected inflation that moves workers up the productivity ladder is dominated by the deadweight loss induced by increased search effort.

1.1 Related Literature

Faccini and Melosi 2023 and Karahan et al. 2017 show that the on-the-job search rate and the rate of job-to-job transitions predict future inflation. Given that on-the-job search effectively generates nominal wage growth at the aggregate level, workers may rationally view search for new work as a way to obtain a nominal raise for themselves. On-the-job search generates wage increases in two ways. A worker can accept an outside offer that dominates her current position or use outside offers to obtain a counteroffer at her current job. In the second case, she remains in the same position but at a higher wage. Pilossoph and Ryngaert 2023 show that higher inflation expectations and realizations are associated with on-the-job search and job-to-job transitions and proposes a model in which workers perceive that offered wages adjust more readily with inflation than existing wages adjust. In this case, expected and realized inflation facilitate raises primarily via labor market transitions. The current paper endogenizes job search to the current price level to study the effects of realized inflation on job-to-job transitions and wage growth for job stayers.

Moscarini and Postel-Vinay 2022 highlight the importance of understanding the origins of nominal wage changes: job-to-job transitions or counteroffers to prevent the poaching of employees. The former, they argue, result from the reallocation of employees from less-productive jobs to more-productive jobs. These raises are not inflationary because firms realize productivity gains that offset the higher wages they pay. Offers that prompt renegotiation at the current firm are inflationary marginal cost shocks for the firms as they must pay a worker more even as productivity remains unchanged or lose that worker. Importantly, Moscarini and Postel-Vinay 2022 assume an exogenous arrival rate of offers. Faccini and Melosi 2023 allow the arrival rate of offers to change with the rate of on-the-job search and show that an increase in the rate of job-to-job transitions is theoretically consistent with wage pressure as it provides a measure of the competition between firms for workers. We allow for the endogenous response of on-the-job search to inflation, showing that inflationary shocks will increase both job-to-job transitions and counteroffers simultaneously and suggesting a potential mechanism for so-called wage-price spirals in which prices and wages increase in response to one another (Blanchard 1986).

This paper contributes to the literature on estimating and explaining the passthrough of price inflation to wage inflation. Hajdini et al. 2022, Jain, Kostyshyna, and Zhang 2022, and Buchheim, Link, and Möhrle 2023 find evidence that the perceived passthrough of inflation to wage growth is low. Pilossoph and Ryngaert 2023 provide evidence that this prompts employed workers to search and potentially speed the arrival of negotiations. Buchheim, Link, and Möhrle 2023 show that - among German workers and firms - expected passthrough increases when workers and firms anticipate negotiations to take place. Higher inflation expectations do not, however, increase the likelihood that German workers ascribe to entering into negotiations. The current paper combines nominal wage rigidity and endogenous search effort to evaluate the mechanisms of passthrough of realized inflation to wage growth.

From a more theoretical perspective, our paper takes the canonical wage negotiation frameworks from the search literature (Postel-Vinay and Robin 2002, Cahuc, Postel-Vinay, and Robin 2006, Lise and Robin 2017, Jarosch 2023), which are developed in real terms, and explicitly adds nominal wage rigidity. Blanco et al. 2022 similarly think about a search framework with nominal rigidities, but do not consider search on-the-job or the wage renegotiation mechanism. As highlighted in Moscarini and Postel-Vinay 2017, this is an important channel for thinking about wage-price spirals, as wage increases on the job are considered cost-push shocks at the firm level.

2 Model

We now outline a model of search on- and off-the-job in an economy with exogenous aggregate productivity z which is growing deterministically at rate g. The price level p in the economy is taken as exogenous and grows at rate g_p . There is a unit mass of firms, each with a vacancy, indexed by their productivity $y \in (\underline{y}, \overline{y})$. The exogenous distribution of vacancies across firms y is denoted by F(y).¹ Per-period output between a worker and a firm of type y is given by Y(z, y).

Workers are homogeneous, infinitely lived, and of measure one, with linear preferences over a single final consumption good given by $u(c_t) = c_t$. They can either be employed or unemployed,

¹In the model we outline below, it is theoretically straightforward to introduce endogenous vacancy creation. However, with endogenous search effort, the vacancy creation decision becomes a function of the distribution of workers across firms and the unemployment rate, which substantially increases the computational burden of the model. We therefore assume vacancies are exogenous from the outset, but allow for the distribution of vacancies to move with the aggregate state exogenously.

and we denote those states by $i \in \{e, u\}$, respectively. Both employed and unemployed workers make search effort decisions $s \in (0, \bar{s}_i)$, which determine the rate at which they meet available vacancies. For employed workers, the rate is $\lambda_e + s$ and for the unemployed it is $\lambda_u + s$. The real cost of search is given by c(z, s), which is increasing and convex in search effort, $\frac{\partial c(z,s)}{\partial s} > 0$, $\frac{\partial^2 c(z,s)}{\partial s \partial s} > 0$. Workers exogenously separate at rate δ from their jobs, and earn a flow value of unemployment B(z). All agents discount the future at rate β .

2.1 Wage Setting and Wage Contracts

When a worker and a firm meet, they decide on an initial nominal wage w, and agree that it will grow at rate $(1 + g)(1 + g_p)$ in the absence of any events which change its trajectory (i.e., outside offers). The fact that the nominal wage will grow with the price level mimics the idea of a "cost of living adjustment" (COLA), so that real wages stay fixed over the course of the match. The fact that wages also grow with aggregate TFP implies that the labor share remains constant in the absence of outside events such as offers which can adjust the worker's bargaining position.

What kind of events can change the path of wages? The wage contract (its level) will be renegotiated only by mutual consent; this can happen if a worker receives an outside offer which dominates the wage they currently make, forcing the firm to adjust the base pay upward, if it is feasible. In this case, wages are determined by Bertrand competition between the incumbent firm and the poaching firm (Postel-Vinay and Robin 2002). In some cases, the incumbent firm will not be able to provide a wage which dominates the new offer, and the worker will leave to the new firm. The wage they receive there will also be determined by Bertrand competition between the incumbent firm and the poaching firm.

When firms meet unemployed workers, they make take-it-or-leave-it (TIOLI) offers. The value of unemployment to a worker can thus be written as:

$$U(z) = \max_{s \in [0,\bar{s}_u]} B(z) - c(s,z) + \beta (1 - (s + \lambda_u)) U(z') + \beta (\lambda_u + s) \int_y \max \left\{ U(z'), W(w, y, p', z') \right\} dF(y) = B(z) + \beta U(z')$$

The second equality comes as a direct result of the TIOLI offers assumption: firms will offer workers a wage that is just large enough so that they are indifferent between taking the offer and remaining unemployed; as such , they receive the same value regardless, so that the future value collapses to a simple expression. Because workers get the same value whether or not they search and search is costly, all unemployed workers choose $s_u^* = 0$. While we know that this is not an accurate description of the search behavior of workers (in fact, the definition of an unemployed worker is that they are actively searching), our focus is on the search behavior of the employed, so we make this simplifying assumption.²

Turning to the employed, consider a worker who is currently with a firm of type y_1 making a nominal wage w which is contracted to grow at rate $(1 + g)(1 + g_p)$ when the current state is p, z that is contacted by y_2 . Bertrand competition between the employers will lead to one of the three following outcomes:

Case 1 No outbidding, contract remains the same. The worker will not be able to use the outside offer to change their current contract if $y_2 < y_1$ and

$$W\left(z'y_2, y_2, p', z'\right) \le W\left(w', y_1, p', z'\right)$$

where $w' = w (1 + g) (1 + g_p)$. Why? Because the maximum wage the poaching firm can offer to start is $z'y_2$, that is the total output the pair will produce next period. Since this provides a value to the worker which is lower than the value they receive by remaining at the current firm where their wage will grow to w', the worker has no credible threat, and their contract remains unchanged.

Case 2 No outbidding, wage contract is renegotiated to ϕ^{reneg} . Suppose that $y_2 < y_1$, but instead:

$$W(z'y_2, y_2, p', z') > W(w', y_1, p', z')$$

In this case, the worker indeed has a credible threat since the poaching firm is able to offer $z'y_2$ and make the worker better off. The incumbent firm then must offer a wage, call it ϕ^{reneg} , which will make the worker indifferent between staying and leaving:

$$W\left(\phi^{\text{reneg}}, y_1, p', z'\right) = W\left(z'y_2, y_2, p', z'\right)$$

Case 3 **Poaching firm hires worker at wage** ϕ^{poach} . If $y_1 \leq y_2$, the incumbent firm will be outbidded by the poaching firm. Therefore, the worker moves to the poaching firm, and gets a new wage ϕ^{poach} which satisfies:

$$W\left(\phi^{\text{poach}}, y_2, p', z'\right) = W\left(z'y_1, y_1, p', z'\right)$$

The above cases imply that there is a cutoff firm, call it $q(w, y_1, p, z)$, which is the lowest

²The assumption can easily be relaxed by either allowing workers to receive some value in employment that is a share of the value of unemployment, or by making the value of unemployment depend on the price level through a benefit which is given nominally. In both cases searching has a positive return.

productivity firm that triggers a wage renegotiation or a job-to-job transition for a worker at firm y_1 employed at wage w. This firm is defined implicitly as the firm q such that:

$$W(w', y_1, p', z') = W(z'q, q, p', z')$$

As we show graphically in Figure 1, if a worker gets an offer from a firm $y_2 < q(w, y_1, z, p)$, nothing happens. Above this value, but below the current firm y_1 , a wage renegotiation gets triggered without a job-to-job transition. Above this value, the worker moves to the new firm.

Importantly, this cutoff firm depends not only on the firm the worker is employed in, but also on their current nominal wage. Specifically, for a given price level, the lower the current nominal wage, the lower is the cutoff firm, implying that a larger set of firms will induce a wage renegotiation. To pre-empt the exercises we will perform which shock the rate of inflation, a higher price level lowers the real wage, which lowers the cutoff firm $q(\cdot)$.

$$\underbrace{\underbrace{y}}_{\text{Status Quo}} \underbrace{q(w, y_1)}_{\text{Reneg.}} \underbrace{y_1}_{y_1} \underbrace{y_2}_{y_1}$$

Figure 1: Offers determining renegotiation and poaching.

With these cases in mind, we can now write the value of employment to a worker with current nominal wage w employed at firm y:

$$\begin{split} W(w, y, p, z) &= \max_{s \in [0, \bar{s}_e]} \frac{w}{p} - c\,(s, z) + \beta \delta U\,(z\,(1+g)) \\ &+ \beta\,(1-\delta)\,(s+\lambda_e) \int^{q(w, y, p, z)} W\,(w', y, p', z')\,dF\,(x) \\ &+ \beta\,(1-\delta)\,(s+\lambda_e) \int^y_{q(w, y, p, z)} W\,(\phi^{\text{reneg}}\,(y, x, p', z')\,, y, p', z')\,dF\,(x) \\ &+ \beta\,(1-\delta)\,(s+\lambda_e) \int^{\bar{y}}_{y} W\,(\phi^{\text{poach}}\,(x, y, p', z')\,, x, p', z')\,dF\,(x) \\ &+ \beta\,(1-\delta)\,(1-s-\lambda_e)\,W\,(w', y, p', z') \end{split}$$

where

$$p' = p (1 + g_p)$$
$$z' = (1 + g)$$
$$w' = w (1 + g) (1 + g_p)$$

The worker earns a real wage of $\frac{w}{p}$, and pays a search cost c(s, z). With probability δ they separate into unemployment. With complementary probability, they remain employed. If they

receive an outside offer - which happens with probability $s + \lambda_e$, one of the three cases outlined above will happen. Specifically, they will either (i) receive an offer from a firm which does not constitute a credible threat (x < q(w, y, p, z)), and their wage will grow according to the agree upon contract, (ii) receive an offer from a firm which induces a change in the contracted wage to ϕ^{reneg} , but no job-to-job transition $(q(w, y, p, z) < x \leq y)$, or (ii) receive an offer from a firm which induces a job-to-job transition (x > y), where they receive a wage ϕ^{poach} . Finally, the worker may not receive an offer in which case the status quo remains.

The value of employment to the firm is:

$$\begin{split} J\left(w, y, p, z\right) &= Y\left(z, y\right) - \frac{w}{p} \\ &+ \beta \lambda_{e}^{*}\left(w, y, p, z\right) \int_{q(w, y, p, z)}^{y} J\left(\phi_{w}^{\text{reneg}}\left(y, x, p', z'\right), y, p', z'\right) dF\left(x\right) \\ &+ \beta \left[\lambda_{e}^{*}\left(w, y, p, z\right) \int^{q(w, y, p, z)} dF\left(x\right) + (1 - \delta)\left(1 - s_{e}^{*}\left(w, p, z\right)\right)\right] J\left(w', y, p', z'\right) \end{split}$$

where

$$p' = p (1 + g_p)$$
$$z' = (1 + g)$$
$$w' = w (1 + g) (1 + g_p)$$
$$\lambda_e^* (w, y, p, z) = (s_e^* (w, y, p, z) + \lambda_e) (1 - \delta)$$

The firm creates output Y(z, y), but must pay its worker $\frac{w}{p}$. It then takes as given the worker's search behavior, $s_e^*(w, y, p, z)$ which determines the probability that the relationship remains in tact, and the probability that wages will be renegotiated in the case that it does. In the event of a separation, we assume that the firm earns a value of zero.

2.2 Balanced Growth

Proposition 1 If (i) Y(z,y), B(z), c(z,s) are homogeneous of degree 1 in z and (ii) wage contracts are indexed to inflation and TFP, then

$$U\Big((1+g)\cdot z\Big) = (1+g)\cdot U(z)\,,$$

$$W\Big((1+g)(1+g_p)\cdot w, y, (1+g_p)\cdot p, (1+g)\cdot z\Big) = (1+g)\cdot W(w, y, p, z), \text{ and} \\J\Big((1+g)(1+g_p)\cdot w, y, (1+g_p)\cdot p, (1+g)\cdot z\Big) = (1+g)\cdot J(w, y, p, z)$$

Proposition 1 implies that the model scales with growth, so that we can solve the model for a single pair z, p. Critically, the rate of inflation does not matter for allocations as long as it is deterministic and wages are indexed appropriately.

3 Calibration

3.1 Parameterization and Targeted Moments

We calibrate the model to match moments from the pre-COVID US economy in the first two decades of the 2000s and set its frequency to be monthly. In Table 1, we display parameter estimates, along with the data source underlying each targeted moment. In Table 2, we display simulated moments together with targeted moments, providing a sense of the fit of the model. In Table 2, we display parameter estimates, along with the data source underlying each targeted moment. We set the discount rate, β , so that it is consistent with an annual interest rate of 5 percent. We set the deterministic TFP growth rate, g, so that it is consistent with an annual growth rate of 0.5 percent, reflecting the average annual growth rate of labor productivity from 2000 through 2019. We set trend inflation, g_p , so that it is consistent with an annual inflation rate of 2 percent. Last, we set the job offer arrival rate of unemployed workers, λ_u , to be 0.31 so that it is consistent with the average UE transition rate over the same time period.

In our quantitative exercise, we allow for the job destruction rate faced by workers to depend on firm productivity. We assume that the relationship is linear such that $\delta(y) = \delta_0 + \delta_1 y$. We target δ_0 to match the empirical separation rate of 0.013, yielding an estimate of δ_0 around 0.014. We target δ_1 to match the separation-wage elasticity of -0.0392 documented by Jung and Kuhn 2019 using the SIPP, yielding an estimate of -0.058.

We assume that the search cost function takes the form $c(s) = c_0 s^{\kappa}$. We calibrate κ to match the search effort-wage elasticity of -0.063 documented by Faberman et al. 2022 using the Survey of Consumer Expectations (SCE), where search effort is captured by the decision to submit a job application. Our estimate for κ is 2.15, implying that search costs are approximately quadratic. We calibrate c_0 to match the relative offer yield ratio between employed and unemployed workers of 0.237 documented by Faberman et al. 2022 using the SCE, finding that c_0 is 60.8. Intuitively, the relative offer yield ratio between employed and unemployed workers is closely related with c_0 , as unemployed workers will never put forth search effort in our model and search effort is decreasing in c_0 for employed workers. We use the average monthly job-to-job transition rate, found to be 0.0241 from 2000 through 2019 by Fujita, Moscarini, and Postel-Vinay 2024, to discipline the job offer arrival rate, λ_e , faced by employed workers. Our estimate for λ_e is 0.058, which is substantial in magnitude, reflecting the fact that employed workers reject the preponderance of outside offers, as most workers are employed at the top of the firm productivity distribution.

We assume that the exogenous vacancy distribution across firm types is a truncated Beta so that $y \sim Beta(\tilde{\alpha}, \tilde{\beta})$, with support [b, 1]. In order to discipline parameters governing the vacancy distribution, we target the average 12-month wage growth of job movers and job stayers ³. The empirical moments we target are measured from administrative payroll data as 0.08 and 0.039 for job movers and job stayers respectively (Grigsby, Hurst, and Yildirmaz 2021). Intuitively, wage dynamics are intimately related with properties the vacancy distribution, particularly the magnitude of dispersion. Our estimate for $\tilde{\alpha}$ is 10.4 and our estimate for $\tilde{\beta}$ is 1.19, which produces a vacancy distribution compressed near the top of the firm productivity ladder.

Finally, to calibrate the flow value of unemployment b, we target the replacement rate of average flow output, $\frac{b}{\mathbb{E}[y]}$, using a target of 0.75 estimated in Chodorow-Reich and Karabarbounis 2016. Given that the wide range of structural estimates of the replacement rate found in the literature, we place comparatively less weight on this target, allowing for wage growth moments to inform b through its influence on the vacancy distribution. This yields an estimate of b near 0.825.

Symbol	Parameter Description	Value	Source/Target		
A. Externally calibrated parameters and normalizations					
β	Discount rate	0.996	Annual interest rate of 5%		
g	Det. TFP growth rate	0.00041	Average TFP growth: 0.5%		
$\check{\lambda}_u$	Job arrival rate, unemployed	0.31	ČPS, 2001-2019		
g_p	Trend inflation	0.0017	CPI/PCE inflation (2000-2019)		
B. Internally calibrated parameters					
κ	Elasticity of search cost	2.15	Search-wage elasticity		
c_0	Search cost parameter	60.8	Relative offer yield ratio		
λ_e	Offer arrival rate of emp.	0.058	EE transition rate		
b	Flow value of unemployment	0.825	Replacement rate		
δ_0	Intercept of job destruction	0.013	EU rate		
δ_1	Slope coefficient of job destruction	-0.058	Separation-wage elasticity		
$ ilde{lpha}, ilde{eta}$	Parameters governing vacancy dist.	10.4, 1.19	Average wage growth of movers, stayers		

Table 1: Calibrated P	arameter Values
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We briefly discuss the fit of the model with targeted moments and untargeted moments of interest. Our model captures the search-wage elasticity and offer yield ratio well, allowing the implied job-to-job transition rate to closely match the empirical moment we target. Our model also comes close to matching the average wage growth of job stayers and job movers, with the average 12 month wage growth of job movers exceeding the average 12 month wage growth of job stayers.

Our model captures the wage dynamics of workers well because the vacancy distribution is compressed near the top of the firm productivity ladder. As a result, the wage distribution

³It is important to mention that the 12-month average wage growth of job movers encompasses both workers who experience a non-employment spell before being hired by a new firm, along with workers who experience a job-to-job transition

is also quite compressed, as depicted in Figure 1, an outcome that departs substantially from the empirical distribution of wages in the US labor market: to be specific, the variance of log real wages is 0.07. Yet, it is important to emphasize that much of this relates to the fact that our model abstracts from worker heterogeneity and attendant sorting patterns between highly productive firms and workers. A large literature finds that only a small fraction of observed wage dispersion reflects firm heterogeneity or dispersion in negotiation rent, which are the only two channels that produce wage dispersion in our model (Bonhomme et al. 2023, Lamadon et al. 2024). As such, wage dispersion in our model is well-aligned with existing evidence.

 Table 2: Targeted Moments in the Data and the Model

Moment	Source	Data	Model
Search-wage elasticity	Faberman et al. 2022	-0.063	-0.061
Offer yield	Faberman et al. 2022	0.237	0.221
EE transition rate	Fujita, Moscarini, and Postel-Vinay 2024	0.0241	0.0238
Replacement rate	Chodorow-Reich and Karabarbounis 2016	0.75	0.841
EU separation rate	CPS (2000-2019)	0.013	0.014
Separation-wage elasticity	Jung and Kuhn 2019	-0.0392	-0.002
Wage growth (job movers)	Grigsby, Hurst, and Yildirmaz 2021	0.039	0.059
Wage growth (job stayers)	Grigsby, Hurst, and Yildirmaz 2021	0.08	0.068



Figure 2: Real Wage Distribution

Note: Figure 1 displays the empirical real wage distribution in steady-state, while also depicting the variance of log real wages.

Using our calibrated model, we plot the search policy function $s(\frac{w}{p}, y)$ for different real wage values at three employers of different productivity levels y in Figure 2a. This highlights the fact that within any firm, search effort is decreasing in the real wage. It also highlights that, as a consequence of the structure of wage determination, returns to search effort are increasing in firm productivity, as the maximum value a worker may receive due to an outside offer is increasing in firm productivity. In Figure 2b, we show that, as a consequence of the introduction of search effort into the framework of Postel-Vinay and Robin 2002, the likelihood of a worker experiencing a job-to-job transition is decreasing in the real wage within any firm, a result that would not obtain if the search effort of workers was taken to be passive.



Figure 3: Search, Job-to-Job Transitions and Wages

Note: Figure 1a considers the relationship between search effort and real wages for workers employed at firms with different productivity levels. Figure 1b considers how the relationship between the likelihood of a job-to-job transition and real wages for workers employed at firms with different productivity levels.

4 The Search Costs of Inflation

4.1 Measuring the Costs of an Unexpected Inflationary Shock

In order to quantify the welfare costs of a large, unexpected inflationary shock, we feed repeated, unanticipated shocks to the rate of inflation to an economy on the balanced growth path. As a baseline, we do not allow agents to adjust their inflation expectations or for wage contracting norms to be adjusted in response to these shocks. We do this to fix attention on the underlying mechanics of our model and the implications they may have for the welfare costs of unanticipated inflationary shocks.

To be specific, at some date τ , we allow the price level to grow at a rate $\hat{g}_p = g_p + \varepsilon$ that is unexpectedly high. As nominal wages *within* a contract are only contracted to grow at rate $(1+g)(1+g_p)$, this results in slower real wage growth as:

$$\frac{(1+g)(1+g_p)}{1+\hat{g}_p} < 1+g$$

We repeatedly feed the ε shock into the inflation rate for three years. In order to illustrate the possible consequences of a very large unanticipated inflationary shock process, we set ε such that it implies an actual annualized inflation rate of 13.9 percent. Then, the force of shock results in annualized inflation exceeding annualized trend inflation by 11 percentage points. After the shock process abruptly ends, its cumulative impact is to increase the price index by 40 percentage points above the anticipated trend. Figure 2 provides a graphical illustration of the shock and Table 3 provides a summary of the shock.

Table 3: Summary of Inflationary Shock

Duration	Magnitude	Actual Inflation Rate	Total Increase
36 months	11.87% (annualized)	13.87% (annualized)	40 percentage points

Note: Table 3



Figure 4: Graphical Illustration of Inflationary Shock

Note: Figure 3a plots the realized price index at time t, normalized relative to the first period of the plot, along with the trend path of prices. Figure 3b plots the inflation rate at different periods, highlighting the magnitude of the shock.

In order to provide a quantitative assessment of the welfare costs induced by the shock process, we simulate a counterpart economy that does not face the shock process. In both economies, we compute the flow value net of search costs w obtained by each worker i in period t in the shocked economy s and the counterpart economy \tilde{b} . Then, for any worker in economy $c \in \{s, \tilde{b}\}$ we have the expression:

$$\tilde{w}_{i,t}^c = \left(\left(w_{i,t}^{real} - c(s_{i,t}) \right) * 1[emp_{i,t} = 1] + b^{real} * 1[emp_{i,t} = 0] \right)$$
(1)

where any worker's *real* flow value is b^{real} if unemployed and given by their real wage $w_{i,t}^{real}$ net of search costs $c(s_{i,t})$ if they are employed. We use this information to consider the welfare costs of the inflation shock to workers on a per-period basis. To do so, we compare the difference in average flow values of workers in the shocked economy and in the baseline economy as as a share of the average flow value:

$$\frac{\mathbb{E}[\tilde{w}_{i,t}^s|t=\tau] - \mathbb{E}[\tilde{w}_{i,t}^{\tilde{b}}|t=\tau]}{\mathbb{E}[\tilde{w}_{i,t}^{\tilde{b}}|t=\tau]}$$
(2)

In Figure 4, we display this measure of the average welfare losses experienced by workers in the shocked economy per period, where the first month plotted is the first month of the shock. In Figure 4a, we may observe that the per-period average welfare losses experienced by workers increase sharply as the shock repeats, with the pace only declining slightly over time, as search effort ramps up and wages become more likely to be renegotiated. When the shock process ceases, as indicated by the dashed line, workers in the shocked economy are receiving nearly 15 percent less in real value than workers in the baseline economy. After the shock is complete, average per-period losses rapidly recover, though there is non-trivial persistence even 7 years after the shock is complete. The main takeaway is that, from the perspective of a worker, our model suggests that large, unanticipated inflationary shocks are very costly to workers.

As shown in Figure 4b, which decomposes the sources of the per-period losses faced by workers into a portion explained by (i) differences in average real wages (ii) differences in average search costs, most of the per-period welfare costs experienced by workers are borne in the form of real wage cuts as seen in the red shaded region, which translate to increased profits for employers. This highlights the redistributive role of unanticipated inflationary shocks. As the real wages of workers fall in response to the shock, they increase search effort as the returns to an outside job offer grow: this mitigates the loss in real wages but comes at a substantial cost to workers, which is a deadweight loss for the economy.



Figure 5: Per-period Welfare Costs of an Inflationary Shock: Workers

Note: Figure 4a plots the difference between the average real wage net of search costs in the shocked economy and the baseline economy in percentage terms. Figure 4b decomposes the sources of this percentage difference into the amount explained by differences in real wages, represented by the red shaded area, and the amount explained by differences in search costs, represented by the blue shaded area.

In order to consider the *aggregate* welfare costs induced by the shock process, we consider a similar expression that measures the real value of flow *output* of a worker net of search costs, \tilde{y} for workers in either economy:

$$\tilde{y}_{i,t}^c = \left(\left(y_{i,t} - c(s_{i,t}) \right) * 1[emp_{i,t} = 1] + b^{real} * 1[emp_{i,t} = 0] \right)$$
(3)

where the real value of any worker's flow output in unemployment is the flow value of unemployment b^{real} and given by the real flow value of output $y_{i,t}$ net of search costs $c(s_{i,t})$ if employed. We use this information to consider the welfare costs of the inflation shock to firms as experienced on a per-period basis. To do so, we compare the difference in average flow output of workers in the shocked economy and in the baseline economy as as a share of the average flow value:

$$\frac{\mathbb{E}[\tilde{y}_{i,t}^s|t=\tau] - \mathbb{E}[\tilde{y}_{i,t}^b|t=\tau]}{\mathbb{E}[\tilde{y}_{i,t}^b|t=\tau]}$$
(4)

In Figure 4, we display this measure of the aggregate welfare losses experienced by agents in the shocked economy per period, where the first month plotted is the first month of the shock. In Figure 4a, we may observe that the per-period average welfare losses experienced by agents increase sharply as the shock repeats, as average search costs ramps up nearly linearly. When

the shock process ceases, as indicated by the dashed line, agents in the shocked economy are receiving over 1 percent less in real value than agents in the baseline economy. After the shock is complete, average per-period losses rapidly recover, with nearly trivial differences between the baseline economy and the shocked economy a few years later. The main takeaway is that large, unanticipated inflationary shocks are quite costly to the economy as a whole, with the search costs of inflation dominating the reallocation channel.

As shown in Figure 4b, which decomposes the sources of the per-period losses faced by agents into a portion explained by (i) differences in real average match output (ii) differences in average search costs (iii) differences in unemployment benefits, explained by differences in employment rates, most of the per-period welfare losses are, indeed, experienced by agents in the form of increased search costs as seen in the blue shaded region. This highlights the fact that the per-period search costs of inflation are quite large, accounting for 1 percent of value. The reallocation of workers towards better firms that occurs due to increased search effort only results in a small increase in real output. This takes place due to two distinct channels. First, as workers are expending more search effort, they move up the job ladder, so average productivity of matches increases. Second, as job destruction rates are decreasing in firm productivity, the employment rate increases slightly, further increasing output. The second channel implies that the role of home production in the shocked economy is smaller than in the baseline economy. As the ratio of the flow value of unemployment to average match output is quite high, this results in only small output increases. Therefore, on net, the reallocative role of the inflationary shock is trivial in comparison to the role of the search costs of inflation.



Figure 6: Aggregate Per-period Welfare Costs of an Inflationary Shock

Note: Figure 5a plots the difference between the average output per worker net of search costs in the shocked economy and the baseline economy in percentage terms. Figure 5b decomposes the sources of this percentage difference into the amount explained by differences in real output, represented by the red shaded area, the amount explained by differences in search costs, represented by the blue shaded area, and the amount explained by differences in home production, represented by the purple shaded area.

In order to consider the *lifetime* welfare costs of the inflationary shock, we may compute the present discounted value (PDV) of worker flow values net of search costs for each worker, W_i , in both the shocked economy and baseline economy, from the first period of the shock onward:

$$\mathcal{W}_{i}^{c} = \sum_{t=\tau}^{T} \beta^{t} ((w_{i,t} - c(s_{i,t})) * 1[emp_{i,t} = 1] + b * 1[emp_{i,t} = 0])$$
(5)

Then, a natural measure of the lifetime welfare costs of the inflationary shock borne by workers is given by the difference in *average* lifetime welfare across workers, expressed as the share of the average PDV in the baseline economy:

$$\frac{\mathbb{E}[\mathcal{W}_i^s - \mathcal{W}_i^b]}{\mathbb{E}[\mathcal{W}_i^b]} \tag{6}$$

Similarly, we may compute the PDV of real output values net of search costs produced by each worker, \mathcal{Y}_i , from the first period of the shock onward:

$$\mathcal{Y}_{i}^{c} = \sum_{t=\tau}^{T} \beta^{t} ((y_{i,t} - c(s_{i,t})) * 1[emp_{i,t} = 1] + b * 1[emp_{i,t} = 0])$$
(7)

This allows us to compute the aggregate welfare costs of the inflationary shock, which is given by the difference in the *average* discounted stream of output across workers in the two economies, expressed as the share of the average PDV in the baseline economy:

$$\frac{\mathbb{E}[\mathcal{Y}_i^s - \mathcal{Y}_i^b]}{\mathbb{E}[\mathcal{Y}_i^b]} \tag{8}$$

Table 4 summarizes the average lifetime welfare losses induced by the inflationary shock process. The average PDV of workers in the shocked economy is over 2 percentage points lower than the PDV of workers in the baseline economy, with around 90 percent of this difference being experienced in the form of lower real wages and 10 percent being experienced in the form of real search costs. As lower real wages result in higher employer profits, the average aggregate welfare losses induced by the inflationary shock are around 0.15 percentage points of the PDV of real output in the baseline economy, with search costs dominating the small increase in output in the shocked economy induced by increased search effort. While this welfare cost appears to be quite small, it is important to emphasize that this is the lifetime welfare effect of a transitory shock. Furthermore, workers in our economy are risk neutral: risk aversion may amplify aggregate lifetime welfare losses substantially.

Table 4: Lifetime welfare consequences of shock

	Change
Worker welfare changes (share of baseline)	-0.0219
Wage channel	-0.02
Search cost channel	-0.019
Overall welfare changes (share of baseline)	-0.0015
Output channel	0.0002
Search cost channel	-0.0016
Unemp flow value channel	-0.0001
enemp: now varue enamer	0.0001

Note: Row 1 depicts the value of expression 6. Subrows decompose this difference in PDVs into a component explained by the difference in PDV of real wages and a component explained by the difference in PDV of search costs. Row 2 depicts the value of expression 8. Subrows decompose this difference in PDVs into a component explained by the difference in PDV of real output and a component explained by the difference in PDV of search costs.

5 Conclusion

TBD.

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.1 Model Solution

The model can be solved in the following steps:

- 1. For a given home production function $b(\Omega)$ and output functions f(z, y) discount rate β , exogenous separation probability δ , the distribution of vacancies for all states $v(y, \Omega)$, the meeting rates for all states $\lambda(\Omega)$, exogenous search intensities for all states $s(\Omega)$, and stochastic transition matrices for z and p, T(z, z') and $T_p(p, p')$, solve for the surplus function $S(y, \Omega)$ as the unique solution to Equation ??. This implies a solution for $U(\Omega)$ and $M(y, \Omega)$.
- 2. Given some initial values for u_0 and $h_0(y)$, a sequence of stochastic productivity shocks $\{z_t\}_{t=0}^T$ and price level realizations $\{p_t\}_{t=0}^T$ imply a unique path for the unemployment rate, and the distribution of employed workers across firms:

$$\left\{u_t, h_t\left(y\right)\right\}_{t=0}^T$$

- 3. Given the path for the above objects, we can now turn to the dynamics of wages. To solve for wages, given some initial $\{z_0, p_0, u_0, h_0(y)\}$:⁴
 - (a) Construct a grid of wage outcomes, $w^{j}(y, p, y_{l}, p_{l}, z, z_{l})$, where j refers to the iteration of the solution algorithm.
 - (b) Guess an initial value function for $W(y, p, y_l, p_l, z, z_l)$.
 - (c) Construct $\sigma(y, p, y_l, p_l, z, z_l)$, the implied share of surplus the nominal wage $w^j(\cdot)$ generates for the worker.
 - (d) Construct $c_e^*(y, p, y_l, p_l, z, z_l)$, the cutoff search cost value.
 - (e) Iterate on $W(y, p, y_l, p_l, z, z_l)$ using Equation ?? until convergence.
 - (f) Given the updated value $W(y, p, y_l, p_l, z, z_l)$, we can solve for wages for those coming out of unemployment which must satisfy $W(y, p, \emptyset, p, z, z) - U(p, z) = 0 \quad \forall p, z.^5$ We can also solve for wages for any worker transitioning from one firm y to another (equal or higher surplus firm) y' when the state is Ω as $W(y', p, y, p, z, z) - U(z) = S(y, z) \quad \forall p, z$. As $W(y', p, y, p, z, z) = \frac{w(y', p, y, p, z, z)}{p} + W_{cont}(y', p, y, p, z, z),$ $w(y', p, y, p, z, z) = p * (U(z) + S(y) - W_{cont}(y', p, y, p, z, z))$
 - (g) In cases where $p \neq p_l, z \neq z_l, w(y', p, y, p_l, z, z_l) = p_l * (U(z_l) + S(y, z_l) W_{cont}(y', p, y, p, z, z))$ with the restriction that $\max_{y,y_l,z_l,p_l}(w(y', p, y, p_l, z, z_l)) = p * (U(z) + S(y, z) - W_{cont}(y', p, y, p, z, z)))$ and $\min_{y,y_l,z_l,p_l}(w(y', p, y, p_l, z, z_l) = p * (U(z) - W_{cont}(y', p, y, p, z, z))).$

⁴The easiest is to begin with everyone in unemployment, so that the surplus shares are irrelevant.

⁵The \emptyset notation refers to the state of unemployment.

This says that aggregate shocks z, p may force a renegotiation of the wage contract in the following two cases: (1) absent renegotiation, the contracted wage would result in the employer laying off the worker (2) absent renegotiation, the contracted wage would result in the worker quitting into unemployment.

(h) Given this new wage grid, return to (c) and repeat steps (c)-(d) until convergence.

A Additional Figures